

Group members:

Warm-up: in the space below, write down a formula for the area between two curves over an interval $[a, b]$. What do you need to know about the functions describing the curves in order to write the formula? How does the formula reflect your intuition about computing areas?

Problem 1. (Q20 from Lecture 2.1) Erica and Carter were asked to compute the area enclosed by $y = 4x$ and $y = x^3$. They agree that $4x = x^3$ when $x = -2$ and when $x = 2$. Erica thinks the area is

$$\int_{-2}^2 (4x - x^3) dx$$

Carter thinks it is

$$\int_{-2}^2 (x^3 - 4x) dx$$

(a) Who is correct?

(b) How do you think the mistake could reasonably have happened, and how can you avoid it?

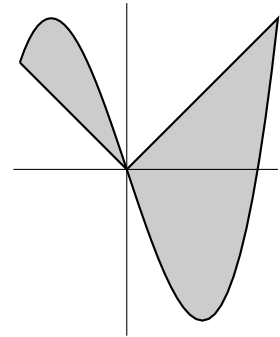
Problem 2. Let A be the region enclosed by $y = x$ and $y = x^3$.

1. Sketch the region A .
2. Set up, but do not evaluate, an integral in terms of x that gives the area of A .
3. Set up, but do not evaluate, an integral in terms of y that gives the area of A .
4. Find the total area of this region.

Problem 3. Let A be the region enclosed by $y = x^2$ and $y = 2x$.

1. Sketch the region A .
2. Set up, but do not evaluate, an integral in the variable x that yields the area of A .
3. Set up, but do not evaluate, an integral in the variable y that yields the area of A .
4. Evaluate one of your integrals to find the area of A and check your answer for plausibility.
5. Suppose that a metal plate has the shape of the region A , assuming that units along the coordinate axes are in feet. If the density of the plate is $1/2$ pound per square foot, then what is the weight of the plate?

Problem 4. Set up an integral formula that yields the area shaded below. The curves determining the boundary of the region are $y = |x|$ and $y = x^3 - 3x$.



Problem 5. (Q32 from Lecture 2.1) Suppose you are given that for all x :

- $f'(x) > 0$
- $g'(x) < 0$

We approximate the area between $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ by rectangles, letting x_i^* be the right endpoints of each subinterval. What can we say about whether the approximation will overestimate or underestimate the true area?