Group members:

Warm-up: what properties must a function $f(x)$ satisfy in order to be a probability density function for a continuous random variable? Compare each of these to the discrete case.

Problem 1. Find a formula for the probability density function $f(x)$ of a uniform random variable on the interval $[1, 4]$. Then compute the median and expected value of $f(x)$. Is this what you expect?

Problem 2. (Lecture 2.6, Q6) One of the following probability questions is different from the others. Explain why.

(a) If you spin a prize wheel 3 times, what is the probability that my winnings add up to exactly \$80?

(b) If you flip two weighted (unfair) coins, what is the probability that exactly one of them comes up tails?

(c) If you pick a random person, what is the probability that her height is exactly 68 inches?

(d) If I spin a wheel of names, what is the probability that it takes exactly 7 spins to land on my own name?

Problem 3. Suppose the prize wheel in part (a) of Problem 2 has 3 different options for the needle to land on: \$0, \$40 and \$80. Compute the probability of winning exactly \$80 in 3 spins. What are the expected winnings?

Problem 4. Suppose we model the wheel of names in part (d) of Problem 2 using an exponential distribution $f(x) = 0.07e^{-0.07x}$ on $[0, \infty)$. The *continuous* random variable X with this distribution measures the event that I spin my name on the wheel after x spins, and I expect to spin my name 7 times every 100 spins, so that $\lambda = 0.07$. What is the probability that it takes exactly 7 spins to land on my name for the first time? Hint: we can compute this as $P(6.5 \leq X \leq 7.5)$.

Problem 5. Let $f(x) = \dfrac{k}{\sqrt{x}}$.

(a) Find the value of k that makes $f(x)$ a probability density function on the interval $[0, 1]$.

(b) Compute the probability that $X \geq \frac{1}{2}$ where X is a continuous random variable on $[0, 1]$ with distribution $f(x)$.

Problem 6. Let X be a continuous random variable on $[0,4]$ with the following probability density function:

$$f(x) = \begin{cases} 1 - x, & 0 \le x < 1 \\ \frac{1}{36}(x^2 - 1), & 1 \le x \le 4. \end{cases}$$

Find the following probabilities.

(a) $P(0 \le X \le 4)$, i.e. verify that $f(x)$ is actually a probability density function on $[0,4]$

(b) $P(X \le 1)$

(c) $P(X \le 2)$

(d) $P(X \le 2 \text{ OR } X \ge 3)$

Problem 7. Let X be a continuous random variable on $[0, 1]$ with probability density function $f(x) = kx^4(1 - x)$.

(a) Find k.

(b) Find the expected value of X.

(c) Find the median of X.

Problem 8. Let X be a continuous random variable on $[1, 4]$ with probability density function $f(x)$. Suppose $\int_1^4 x^2 f'(x)\,dx = 11$, $f(1) = 2$ and $f(4) = 1$. Find the expected value $E[X]$. Hint: integrate by parts.