Group members:

Warm-up: write the most general formula for a power series. Then, with your group, discuss how to find its radius and interval of convergence.

Problem 1. Find the radius and interval of convergence of the following power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n(n+1)}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2}{3}\left(\frac{5x}{4}\right)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (Remember that $0! = 1$ by definition. Also, does this series look familiar?)

Recall the following theorem from the lecture:

Theorem. For a power series $\displaystyle\sum_{n=0}^{\infty} a_n(x - a)^n$, define its **radius of convergence** by

$$R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right|.$$

(Notice this is the *reciprocal* of the limit in the Ratio Test.) Then:

(1) If $R = 0$, the power series only converges at $x = a$.

(2) If $R > 0$ is a finite number, the power series converges for x on the interval $(a - R, a + R)$.

(3) If $R = \infty$, the power series converges for all x on the interval $(-\infty, \infty)$.

WARNING: (2) tells you nothing about the endpoints of the interval $(a - R, a + R)$, so in principle you will have to check these by hand, using series tests.

Problem 2. Suppose the series $\sum_{n=0}^{\infty} c_n 5^n$ converges and the series $\sum_{n=0}^{\infty} c_n 6^n$ diverges. What can be said about the following series?

(a) $\displaystyle\sum_{n=0}^{\infty} c_n 3^n$

(b) $\displaystyle\sum_{n=0}^{\infty} c_n (-7)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} c_n (-5)^n$

Problem 3. For each of the following series, determine (i) the radius of convergence, (ii) the interval of convergence (including testing at endpoints) and (iii) if possible, a formula for the power series on its interval of convergence.

(a) $\displaystyle\sum_{n=0}^{\infty} n! x^n$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3x)^n}{4^{n-1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} n^3 (x-1)^n$

(d) $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^n}{\sqrt{n+1}}$

Problem 4. Use geometric series to represent each of the following functions as a power series, then find its radius of convergence.

(a) $f(x) = \dfrac{1}{1 + 3x}$

(b) $f(x) = \dfrac{7}{1 + x^2}$

(c) $f(x) = \dfrac{x^2}{4 - x^2}$

Problem 5. Find a power series representation for $f(x) = \ln(5 + x)$ centered at $x = 0$. What is its radius of convergence?