

Group members:

Warm-up: sketch a few levels curves of the multivariable function $f(x, y) = x^2 + y^2$. Discuss with your group what the full 3-dimensional graph of $f(x, y)$ looks like.

Problem 1. (Lecture 4.2, Q50) Consider the implicit equation $zx = y$.

(a) Rewrite this as an explicit function $z = f(x, y)$.

(b) What is the domain of f ?

(c) Solve for and sketch a few level sets of f . What do the level sets tell you about the graph $z = f(x, y)$?

Problem 2. (Lecture 4.1, Q34) Gabby is trying to find the equation of a plane P , but she doesn't know any points on the xz -plane or yz -plane. Instead she knows that P contains the points:

$$A = (1, 3, 6) \quad B = (5, 3, 4) \quad C = (7, 5, 10)$$

Using points A and B , she decides that $m_x = \frac{4-6}{5-1} = -\frac{1}{2}$. Using points A and C , she decides that $m_y = \frac{10-6}{5-3} = 2$.

(a) Which of Gabby's conclusions do you agree with and which do you disagree with? Why?

(b) How could you fix the one that is wrong?

Problem 3. Describe the level curves $f(x, y) = k$ of the function $f(x, y) = \sqrt{x + y}$. Are they circles, ellipses, lines, parabolas, etc.? How are they spaced?

Problem 4. Draw the level curves $f(x, y) = k$ where $f(x, y) = \left(\frac{1}{2}y + x\right)^3$ and $k = -1, 0, 1, 8$.

Problem 5. Sketch the domain of $f(x, y) = \ln(9 - x^2 - y^2)$ in the xy -plane.

Problem 6. Let $f(x, y) = xy^2 + x^2y$.

(a) Compute $f(1, 0)$, $f(0, -2)$ and $f(1, -2)$.

(b) Write the formula for the “coordinate curve” $g(x) = f(x, -2)$.

(c) Compute $g'(1)$. What does this value tell us about the graph of $f(x, y)$ at $(x, y) = (1, -2)$?