Group members:

Warm-up: in words, what does it mean to take the partial derivative of $f(x, y)$ with respect to x? with respect to y? How do the values $f_x(a, b)$ and $f_y(a, b)$ related to the graph $z = f(x, y)$ near $(x, y) = (a, b)$?

Problem 1. Compute the partial derivatives of $f(x, y) = x^2 + 4xy + y^3 + 4y$ with respect to x and y.

Problem 2. (Lecture 4.4, Q20) Suppose Jinteki Corporation makes widgets which it sells for \$100 each. It commands a small enough portion of the market that its production level does not affect the demand (price) for its products. If W is the number of widgets produced and C is their operating cost, Jinteki's profit is modeled by

$$P = 100W - C.$$

Since $\frac{\partial P}{\partial W} = 100$ does this mean that increasing production can be expected to increase profit at a rate of \$100 per widget?

Problem 3. Compute the partial derivatives of the function $f(x, y) = \ln(e^{xy} + x^2 + 2y^4 + 1)$.

Problem 4. Let $f(x, y) = x^2 + 2y^2 - 2x$.

(a) Compute the partial derivatives f_x and f_y.

(b) Find all points (x, y) in the domain of f for which f_x and f_y are both 0 and interpret your answer. What does the graph $z = f(x, y)$ look like at these points?

(c) Compute the second-order partial derivatives f_{xx}, f_{yy} and $f_{yx} = f_{xy}$.

Problem 5. Compute all second-order partial derivatives of $f(x,y) = (x+2)(y-1)e^{x^2+y^2}$.

Problem 6. Let $f(x,y)$ be a function of two variables with continuous partial derivatives of all orders. For each of the following, determine whether the statement is TRUE or FALSE and justify your answer.

(a) $f_{xxy} = f_{xyx}$

(b) $f_{xyxy} = f_{yxyx}$

(c) $f_{xxyxxy} = f_{yyxyyx}$

Problem 7. Find f_{xx} and f_{yx} where $f(x,y) = 11 - y + y^4x^2 + 12x^{2022} + y^{2023}$.

Problem 8. For the function $f(x,y) = 10 - 4x^2 - y^2$,

(a) Find an equation for the line that is tangent to the graph $z = f(x,y)$ at $(1,2)$ and parallel to the x-axis.

(b) Find an equation for the line that is tangent to the graph $z = f(x,y)$ at $(1,2)$ and parallel to the y-axis.

Problem 9. Recall that for a function $f(x, y)$ to have $f_{yx} = f_{xy}$ ("equality of mixed partials"), it is necessary for these second-order partial derivatives to be *continuous*. Show that for the function

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

the mixed partials f_{yx} and f_{xy} have different values at $(0, 0)$. Why isn't this a contradiction to Clairaut's Theorem? *Hint: use the limit definitions of f_x and f_y to find piecewise formulas for these first-order partial derivatives before using the limit definitions of f_{yx} and f_{xy} to find the values $f_{yx}(0, 0)$ and $f_{xy}(0, 0)$.*