Group members:

Warm-up: write an equation for the tangent plane T to the graph $z = f(x, y)$ near the point (x_0, y_0). What can you say about the points on T near (x_0, y_0)?

Problem 1. Find an equation for the tangent plane to $z = \ln(2x + y)$ at the point $(-1, 3)$.

Problem 2. Find the linear approximation to $z = 3 + \dfrac{x^2}{16} + \dfrac{y^2}{9}$ at the point $(-4, 3)$.

Problem 3. Find the linear approximation to $z = 4x^2 - ye^{2x+y}$ at the point $(-2, 4)$.

Problem 4. (Lecture 4.5, Q22) Suppose I decide to invest \$10,000 expecting a 6% annual rate of return for 12 years, after which I'll use it to purchase a house. The formula for compound interest

$$P = P_0 e^{rt}$$

indicates that when I want to buy a house, I will have $P = 10,000e^{0.72}$ dollars. I accept that my expected rate of return might have an error of up to $dr = 2\%$. Also, I may decide to buy a house up to $dt = 3$ years before or after I expected.

(a) Write the formula for the differential dP at $(r_0, t_0) = (0.06, 12)$.

(b) Given my assumptions, what is the maximum estimated error dP in my initial calculation?

(c) What is the actual maximum error in P?

(d) If I allow myself some flexibility in the initial investment P_0, how can I express the error dP in terms of the errors of all three variables: P_0, r and t?

Problem 5. For the function $g(x, y) = \sqrt{41 - 4x^2 - y^2}$, approximate $g(2.1, 2.9)$ using the point $(x_0, y_0) = (2, 3)$. How much error is involved in your approximation?

Problem 6. Does the function

$$f(x, y) = \sqrt{x^2 + y^2}$$

have a tangent plane at $(0, 0)$? Why or why not? Interpret this graphically.