Group members:

Warm-up: what do single and double integrals represent in general? How about a triple integral?

**Problem 1.** (Ex. 6.4.7 from Lecture 6.4) Suppose $D$ is the bounded region enclosed between the graph of $y = 4x^2 + z^2$ and the plane $y = 4$. Set up the bounds of the integral $\iiint_D f(x, y, z)\, dV$.

**Problem 2.** (Q36 from Lecture 6.4) Rewrite the integral $\int_0^2 \int_{2-x}^2 \int_0^{4-x^2} f(x,y,z)\,dz\,dy\,dx$ as an integral with the differential $dx\,dz\,dy$.

**Problem 3.** (Q26 from Lecture 6.4) Let $R$ be the region enclosed by $y = \sqrt{25-x^2}$, $z = 6-y$ and $z = \sqrt{y}$. Set up the bounds of $\iiint_R g(x,y,z)\,dV$.

**Problem 4.** (Q14 from Section 6.4) Give the volume of the $x = 2$ cross-section of the region "under" the graph of $w = \frac{z^2\sqrt{13-x^2}}{y}$ and "above" the prism

$$P = \{(x, y, z) : 0 \leq x \leq 3, 1 \leq y \leq 2, -3 \leq z \leq 3\}.$$

**Problem 5.** (Q18 from Section 6.4) Random variables $X, Y$ and $Z$ are **uniform** if their density function has the form

$$f_{X,Y,Z}(x, y, z) = \begin{cases} \frac{1}{V} & \text{if } (x, y, z) \text{ is in } R \\ 0 & \text{otherwise} \end{cases}$$

where $V$ is the volume of $R$. If $X, Y$ and $Z$ are uniform on

$$R = \{(x, y, z) : 0 \leq x \leq 10, 0 \leq y \leq 10, 0 \leq z \leq 10\},$$

compute $P(X \leq 4 \text{ and } Z \geq 3)$.

Here's a review problem from Section 6.3.

**Problem 6.** (Q10 from Section 6.3) Suppose we perform an experiment in which a pair of strangers find an amount of money on the ground. Suppose $X$ and $Y$ are continuous random variables that model the portion of the money (0 = none, while 1 = all) that each person keeps. Any money not kept is turned into the authorities. Suppose the joint density function of $X$ and $Y$ is

$$f_{X,Y}(x,y) = \begin{cases} 24xy & \text{if } x \geq 0, y \geq 0, \text{ and } x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) In a few sentences, interpret what this density function says about which outcomes are likely and which are not. Feel free to include any comments on human nature that you need to get off your chest.

(b) Set up an integral (or integrals) that computes the probability that each person takes at most twice as much as the other.

(c) Evaluate your integral in part (b).