Due Date: Thursday, January 19 at 11:59PM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

Textbook problems: 16.6

Problem 1. (Lecture 1.2, Exercise 1) Let A be a ring. Use the ring axioms for A to show that for all $x \in A$, $0x = 0 = x0$.

Problem 2. Compute the group of units $A^\times$ in each of the following commutative rings A:

(a) $A = \mathbb{Z}/3\mathbb{Z}$

(b) $A = \mathbb{Z}/4\mathbb{Z}$

(c) $A = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

(d) $A = \mathbb{Z}/12\mathbb{Z}$

(e) $A = \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$

(f) $A = \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$

Problem 3. Prove that there does not exist an integral domain A with exactly 6 elements. *Hint: for such a ring A, $(A, +)$ is an abelian group of order 6. What can you say about such a group?*