

Due Date: Thursday, January 26 at 11:59PM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

Textbook problems: 1.5

Problem 1. Find two rings A and B and a function $\varphi : A \rightarrow B$ that is an abelian group homomorphism but *not* a ring homomorphism. Prove all of your assertions.

Problem 2. Let A be a commutative ring of characteristic p , where p is a prime number. Prove that the map $\varphi_p : A \rightarrow A$ defined by $\varphi_p(x) = x^p$ is a ring homomorphism. What can you say about $\varphi_n(x) = x^n$ for other values of $n \in \mathbb{Z}$?

Problem 3. Let X be a set and $\mathbb{P}(X)$ its power set, with ring operations as defined in Lecture 2.1.

- (a) For an element $U \in \mathbb{P}(X)$, what is $-U$ as a subset of X ?
- (b) Show that the characteristic of $\mathbb{P}(X)$ is 2.
- (c) Find the group of units $\mathbb{P}(X)^\times$. Is $\mathbb{P}(X)$ a field?