Due Date: Thursday, February 2 at 11:59PM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

Textbook problems: none this week

Problem 1. (Lecture 3.1, Exercise 1) Prove the Correspondence Theorem for rings: if $\varphi : A \twoheadrightarrow B$ is a surjective ring homomorphism with kernel $K = \ker(\varphi)$, there is a bijective correspondence

$$\left\{ \begin{matrix} \text{ideals } I \subseteq A \text{ with} \\ K \subseteq I \subseteq A \end{matrix} \right\} \longleftrightarrow \{\text{ideals } J \subseteq B\}.$$

Problem 2. For a surjective ring homomorphism $\varphi : A \twoheadrightarrow B$, prove that for any maximal ideal $M \subset B$, the preimage $\varphi^{-1}(M)$ is a maximal ideal of A.

Problem 3. Find all maximal ideals in each of the following rings:

(a) $\mathbb{Z}/6\mathbb{Z}$

(b) $\mathbb{Z}/12\mathbb{Z}$

(c) $\mathbb{Z}/30\mathbb{Z}$

(d) $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$

For 1 point of extra credit, state a conjecture about which ideals in $\mathbb{Z}/n\mathbb{Z}$ are maximal. You do not have to prove your conjecture, but you can!