Due Date: Wednesday, February 23 at 10AM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Canvas before the listed due date.

Textbook problems: Section 2.7 #8, 14, 24, 28; Section 3.1 #10, 12, 14, 18, 22, 26 (all 3 parts!); Section 3.2 #9, 10, 12, 14, 16, 18; Section 3.3 #12, 20, 36, 44, 48

Optional textbook problems: the odd numbered problems from Sections 2.7 - 3.3

Problem 1. The weekly demand for chlorine at a wastewater treatment plant (in thousands of gallons) can be modeled by a continuous random variable C with probability density function

$$f(x) = xe^{-x^2/2} \quad \text{on } [0, \infty).$$

If 2 thousand gallons of chlorine are stocked at the beginning of the week, what is the probability that there will be enough chlorine to meet the weekly demand?

Problem 2. Explain why the following is true:

$$\frac{1}{8\sqrt{2\pi}} \int_{-\infty}^{\infty} xe^{-\frac{1}{2}\left(\frac{x-12}{8}\right)^2} dx = 12.$$

Problem 3. Let $f(x) = \arctan(x)$.

(a) Write down the first 10 nonzero terms of the sequence of Taylor polynomials for $f(x)$ about $a = 0$. (The highest power of x will be 19, so you are computing $T_{19}(x)$.)

(b) Now plug in $x = 1$ to get an estimate for $f(1)$. Use this estimate to get a decimal approximation for π.

(c) How many decimals of π does your approximation reveal? How many decimals would $T_{25}(1)$ agree with?

Problem 4. (EXTRA CREDIT, 10 points) Write a program in Python that accurately determines the first 20 digits of e (after the decimal, e.g. the first two are .71). You may submit this as a Python notebook file on Gradescope under the "HW 3 Extra Credit" assignment. To receive full credit, your notebook should contain a detailed explanation of what your code does in each step and how you can guarantee that all 20 digits are accurate.