**Due Date:** Thursday, September 15 at 5PM EDT

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

**Textbook problems:** 6.2, 7.3, 8.3, 8.5

**Problem 1.** (Lecture 3.1, Exercise 1) Let $a, b \in \mathbb{Z}$ with $\gcd(a, b) = d$.

(a) Verify $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

(b) Given one solution $(x, y)$ to $ax + by = d$, show that every solution is of the form $\left(x + \frac{bk}{d}, y - \frac{ak}{d}\right)$ for some $k \in \mathbb{Z}$.

**Problem 2.** Let $a, b, c \in \mathbb{Z}$.

(a) Suppose $a$ divides $bc$ and $\gcd(a, b) = 1$. Prove that $a$ divides $c$.

(b) Suppose $a$ and $b$ both divide $c$ and $\gcd(a, b) = 1$. Prove that $ab$ divides $c$.

(c) Suppose that $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$. Prove that $\gcd(ab, c) = 1$.

**Problem 3.** How many divisors $d \in \mathbb{N}$ does $n = 1000$ have?

**Problem 4.** (a) Show that there are no $a, b \in \mathbb{N}$ such that $a^2 = 2b^2$.

(b) Use (a) to prove that $\sqrt{2}$ is irrational.

(c) Is $\sqrt{p}$ rational for any prime $p$? Why or why not?