Due Date: Thursday, February 9 at 11:59PM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

Textbook problems: 3.4,

Problem 1. Determine the kernel of the ring homomorphism

$$\varphi : \mathbb{C}[x,y] \longrightarrow \mathbb{C}[z], \quad f(x,y) \longmapsto f(z,z^2).$$

Then find the image of φ. What does the First Isomorphism Theorem say in this case?

Problem 2. In the ring $\mathbb{F}_3[x]$, decide if each of the following ideals is prime, maximal or neither:

(a) (x) (b) (0) (c) (x^2+1) (d) (x^2+2) (e) (x^3-x-2)

Problem 3. In the ring $\mathbb{Z}[x]$, decide if each of the following ideals is prime, maximal or neither:

(a) (x) (b) (2) (c) $(2,x)$ (d) (x^2-1) (e) (x^2+1)

Problem 4. Factor the polynomial x^3-1 into irreducible factors in the polynomial ring $\mathbb{F}_7[x]$. You must prove that each factor you find is irreducible.