Due Date: Wednesday, April 20 at 10AM EDT

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Canvas before the listed due date.

Textbook problems: Section 5.3 #8, 12, 26, 32; Section 5.4 #6, 10, 18, 30, 32, 34; Section 5.5 #8, 18, 20, 22; Section 5.6 #8, 22, 30, 34; Section 5.7 #16, 22; Section 6.1 #6, 8, 24
Optional textbook problems: the odd numbered problems from Sections 5.3 - 5.7 and 6.1.

Problem 1. A hiker visiting a state park is descending a mountain as night falls, decreasing visibility. Their altitude is given by the function

$$h(x,y) = \frac{5(x+y)}{x^2+y^2+1}$$

based on their geographical coordinates (x,y), in miles to the east and north of the park entrance in the southwest.

(a) If the hiker is currently 4 miles north and 5 miles east of the park entrance, in which direction should they head to descend the mountain most quickly?

(b) How steep is the descent in this direction?

(c) Is this in the direction of the park entrance?

Problem 2. The temperature in Lake Charlotte is given by

$$T(x,y) = x^2e^y - xy^3$$

(in degrees Celsius) with $(0,0)$ representing the center of the lake. To stay warm, you swim in a circle along the path described by $x(t) = \cos\left(\frac{t}{10\pi}\right), y(t) = \sin\left(\frac{t}{10\pi}\right)$, where t is in seconds.

(a) Find the rate of change of temperature as a function of time using the chain rule.

(b) Confirm your answer in part (a) by writing T as a function of t and then differentiating with respect to t.

Problem 3. The surface S described by $z = x^2 + y^2$ intersects the plane P with equation $2x + 4y + 20z = 400$ at the point $(4, -2, 20)$. Find the angle of intersection between the tangent plane to the surface at this point and the plane P.

Problem 4. The area of a triangle with side lengths x, y and z is given by

$$A(x,y,z) = \sqrt{s(s-x)(s-y)(s-z)}$$

where $s = \frac{1}{2}(x+y+z)$. Show that a triangle with fixed perimeter has maximum area when it is an equilateral triangle.