

Due Date: Thursday, March 23 at 11:59PM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Gradescope before the listed due date.

Textbook problems: 7.4, 7.6

Problem 1. For $n \geq 2$, let $\zeta_n = e^{2\pi i/n}$ be a primitive n th root of unity. (Here, *primitive* means ζ_n generates the cyclic group of n th roots of unity.)

- (a) Determine the minimal polynomial of ζ_n over \mathbb{Q} for $n = 4, 6, 8, 9, 10, 12$.
- (b) Is $\zeta_5 \in \mathbb{Q}(\zeta_7)$? Explain why or why not.
- (c) For which values of n is $[\mathbb{Q}(\zeta_n) : \mathbb{Q}] \leq 3$?