# Intro + Lecture 12.1

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**Agenda:**

- introductions
- syllabus
- quick tour of Canvas
- intro to vectors

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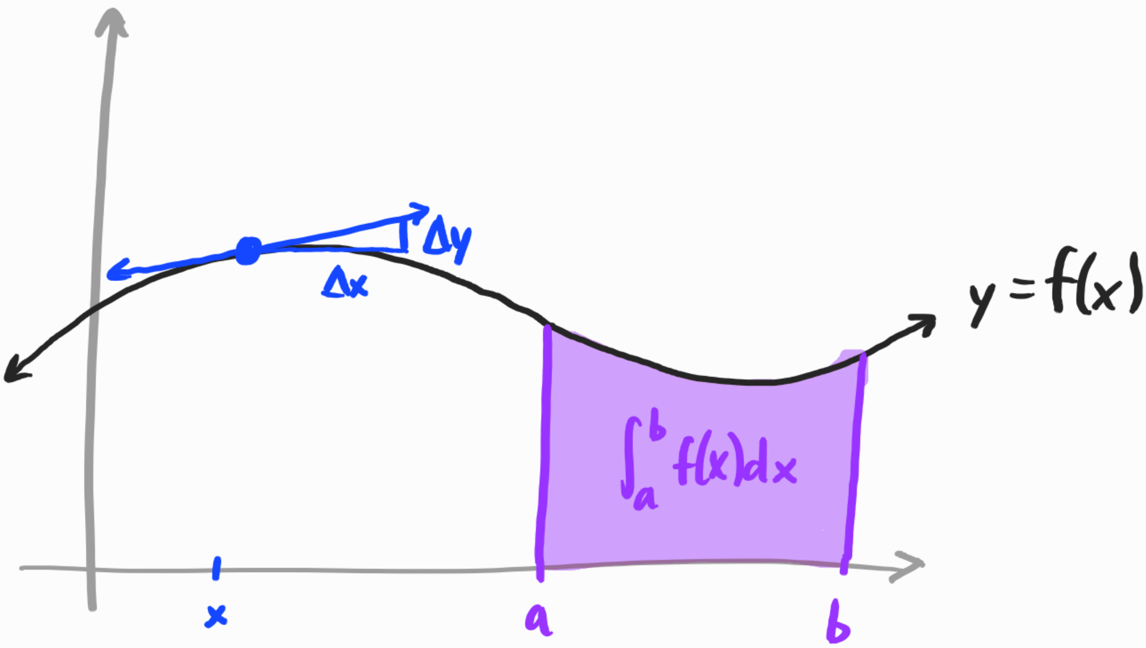
## Introduction

In Calculus I–II, we learned all about measuring change (with derivatives and integrals) for functions of a <u>single</u> variable.



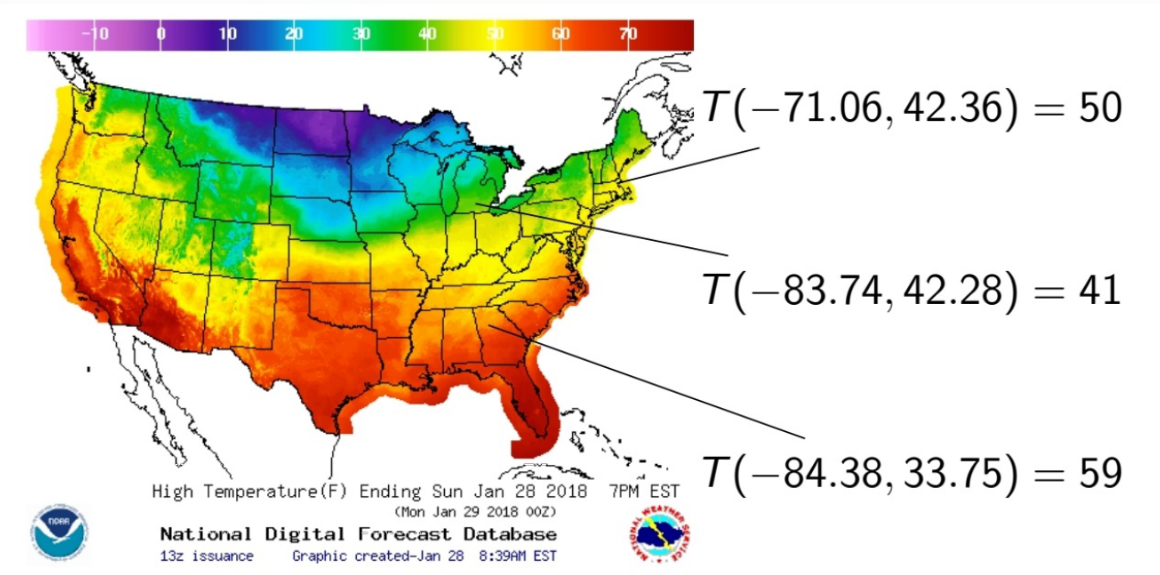

This course is all about functions with <u>multiple</u> input variables,

$f(x,y)$, $g(x,y,z)$, $h(\ldots)$

$f(x,y), \quad g(x,y,z), \quad h(x,y,z,w),$

$j(x_1, \dots, x_{50})$, etc.

$\boxed{\text{Ex}}$ Given a latitude and longitude $(x,y)$, let $T(x,y)$ be the current temperature (°F) at that point on the Earth's surface.




Then we might be interested in how

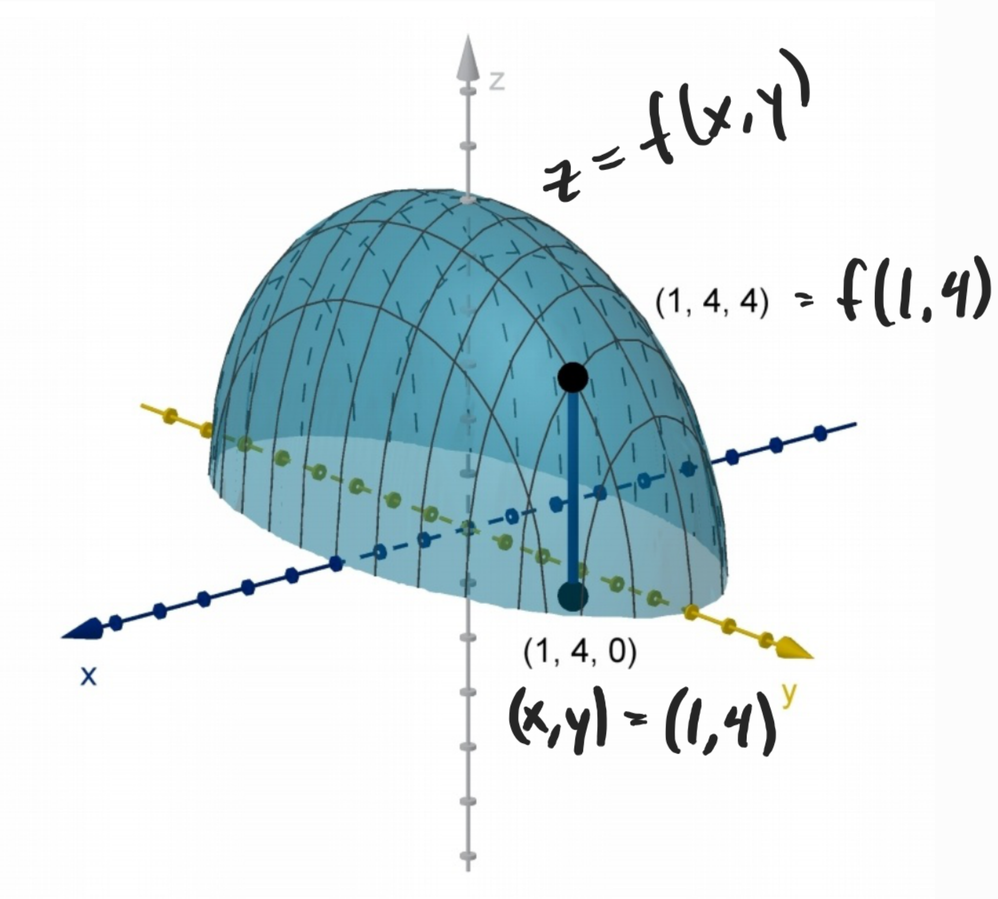
Then we might be interested in how $T(x,y)$ changes as $x$ and $y$ change, possibly independently of each other.

$\boxed{\text{Ex}}$ Though not always realistic, we will mostly focus on functions that are defined algebraically, e.g.

$$f(x,y) = \sqrt{36 - 4x^2 - y^2}.$$

Similar to single variable functions, we can represent a function like th[illegible] graph $z = f(x,y)$ in this as a graph $z = f(x,y)$, in

this case in 3-dimensional space:




Notice that not every point in the xy-plane has a point above it on the graph.

$\boxed{\text{Definition}}$ The **domain** of a multivariable function is the set of input values that have a well-defined output.

$\boxed{\text{Ex}}$ The function

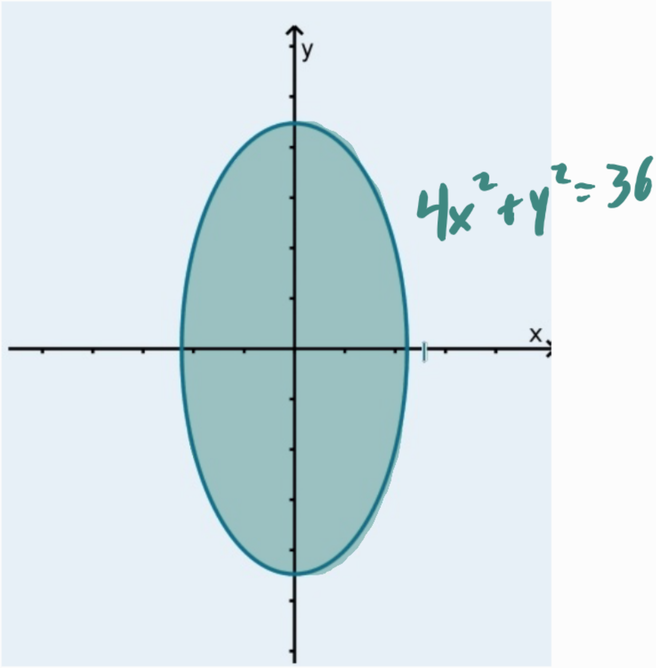
$$f(x,y) = \sqrt{36 - 4x^2 - y^2}$$

is defined when

$$36 - 4x^2 - y^2 \geq 0 \implies 4x^2 + y^2 \leq 36$$

so for all $(x,y)$ inside the ellipse

defined by $4x^2 + y^2 = 36$:

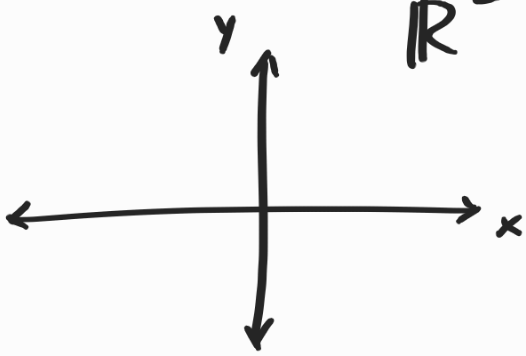


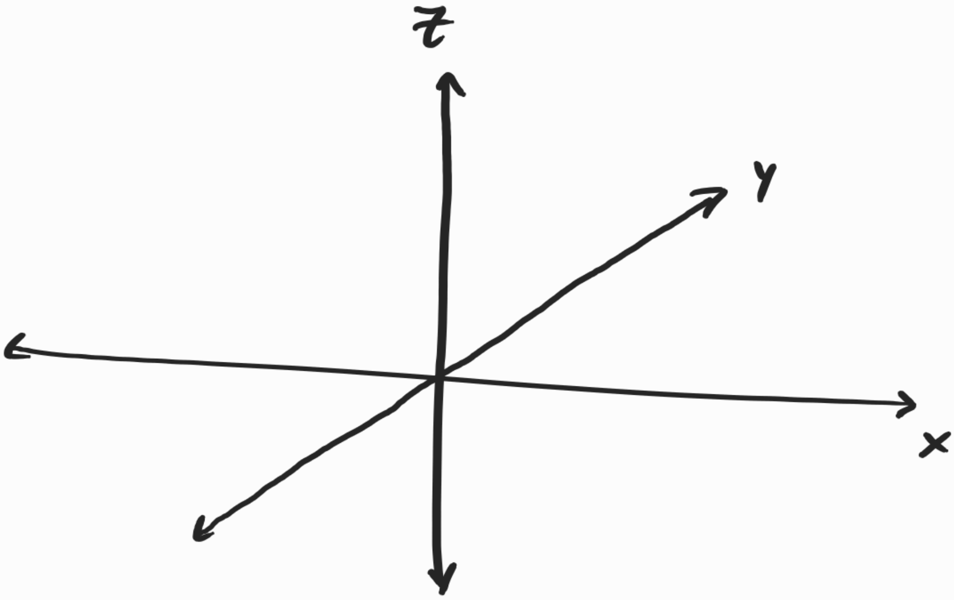

Before we learn more about functions, let's explore Euclidean space a little further.

Notation: n-dimensional coordinate space is written $\mathbb{R}^n$ (the $\mathbb{R}$ is for "real numbers") :

$\mathbb{R} = \mathbb{R}^1$

$\mathbb{R}^2$




$\boxed{\text{Distance Formula}}$ In $\mathbb{R}^n$, the distance between two points

$$A = (x_1, \ldots, x_n) \quad \text{and} \quad B = (y_1, \ldots, y_n)$$

is given by the formula

$$|AB| = \sqrt{(y_1 - x_1)^2 + \dots + (y_n - x_n)^2}.$$

$\boxed{\text{Ex}}$ For $A = (2, -1, 7)$ and $B = (1, -3, 5)$,

$$|AB| = \sqrt{(1-2)^2 + (-3+1)^2 + (5-7)^2}$$

$$= \sqrt{1 + 4 + 4} = 3.$$

Exercise 1: Is $|AB|$ the same as $|BA|$? Why or why not?

Exercise 2: Compute the distance between each pair of points.

(a) $(-2, 3, -1)$ and $(0, 0, 0)$

(b) $(-2, 3, -1)$ and $(2, -3, 1)$

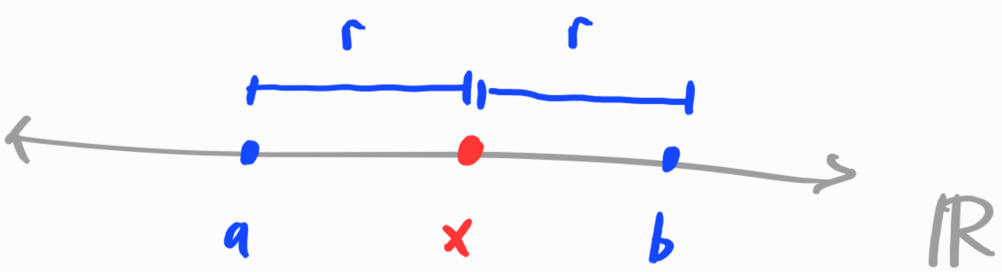
(c) $(0, 4, -1)$ and $(6, 4, 7)$

(d) $(1, 0, 1, 0)$ and $(0, 1, 0, 1)$

With this formula, it is possible to write the equation for a sphere in $n$ dimensions.

**Def** A **sphere** is a set of points which are all a fixed distance away from a single point, called the **center** of the sphere. The fixed distance is called the **radius**.

**Ex** In $\mathbb{R}$, a "sphere" is just two points:

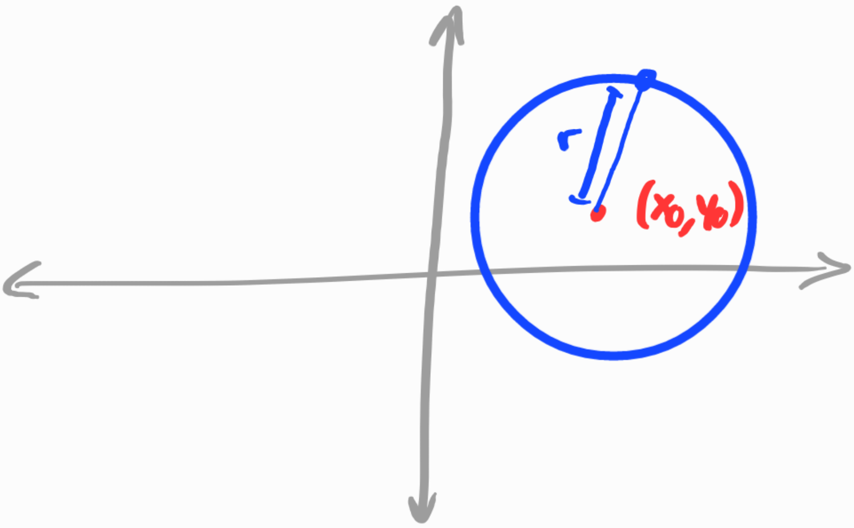



Can you fill in what $r$ and $x$

are? $r =$ ______

$x =$ ______

In $\mathbb{R}^2$, a sphere is just a circle, which can be specified by a point $(x_0, y_0)$ and a radius $r > 0$:



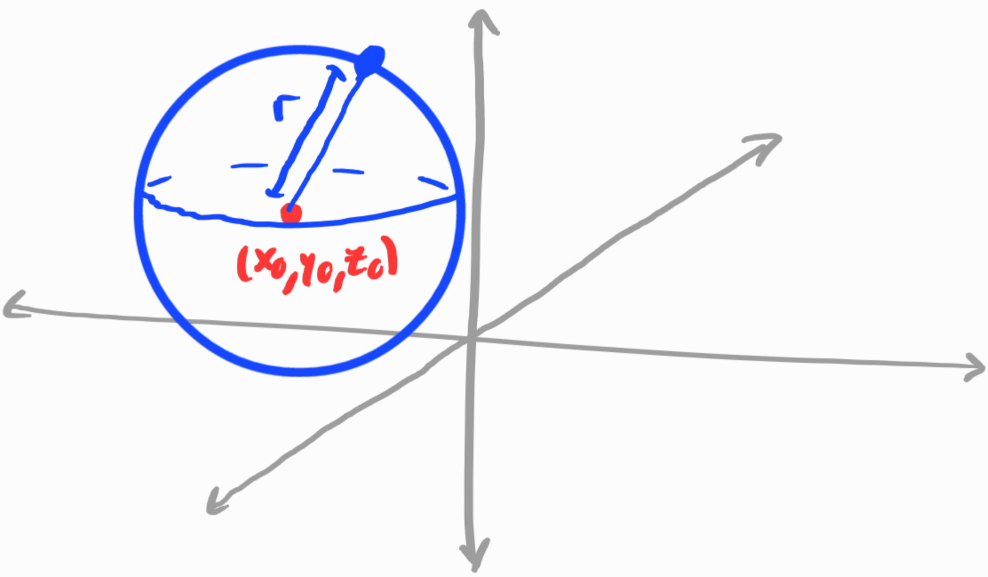

The equation for a circle is

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Can you see how this relates to

Can you see how this relates to the distance formula?

In $\mathbb{R}^3$, a sphere has a center $(x_0, y_0, z_0)$ and a radius $r > 0$:




The equation for a sphere in $\mathbb{R}^3$ is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2.$$

Can you see a pattern?

$\boxed{\text{Formula for a Sphere}}$ In $\mathbb{R}^n$, the sphere with center $P$ and radius $r$ is given the equation

$$|AP|^2 = r^2 \quad (\text{or } |AP| = r)$$

where $A = (x_1, \ldots, x_n)$.

Exercise 3: Write the equation for the sphere with center $(2, -1, 7)$

and radius 6.

Exercise 4: What are the center and radius of the sphere with equation

$$x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0 ?$$

Next time: vectors.