Group members:

Warm-up: on your own, write down the three most challenging concepts you have learned in this class so far. Then, as a group, share some strategies for those types of problems.

As a class: what are the important definitions, formulas, theorems, concepts, techniques, etc. that you should know for the exam.

Problem 1. (CW 2.7-3.1, Problem 3) Compute the first 3 digits after the decimal in the number $e = 2.???\ldots$. Then describe how you would find the first 10 digits.

Problem 2. (HW 3, Problem 3a) Find the degree 19 Taylor polynomial for $f(x) = \arctan(x)$.

Problem 3. (CW 3.2, Problem 5) Make an educated guess at what the sequence $y_n = \sqrt{n^2+n} - n$ converges to. Then verify your guess.

Problem 4. (CW 3.2, Problem 4e) Does the sequence $e_n = \left(1 + \frac{1}{n}\right)^n$ converge or diverge? If it converges, find the limit of the sequence.

Problem 5. (CW 2.6, Problem 8) Let X be a continuous random variable on $[1, 4]$ with probability density function $f(x)$. Suppose $\int_1^4 x^2 f'(x)\,dx = 11$, $f(1) = 2$ and $f(4) = 1$. Find the expected value $E[X]$.

Problem 6. (CW 2.5, Problem 4) There is exactly one constant A for which $\int_0^\infty \left(\frac{2x}{1+x^2} + \frac{A}{x+1}\right) dx$ converges. Find it and evaluate the integral for that constant.

Problem 7. Evaluate the following integral expressions. For any improper integrals, state whether they converge or diverge and, if they converge, what they converge to.

(a) $\int x^2 \sin(x)\, dx$

(b) $\int_1^e \ln(x)\, dx$

(c) $\int e^x \cos(x)\, dx$

(d) $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

(e) $\displaystyle\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx$

(f) $\displaystyle\int (\sin(x)+\cos(x))^2\,dx$

Problem 8. Find all values of p so that $\displaystyle\int_1^\infty \frac{x^p - 1}{x^{2p}}\, dx$ converges.

Problem 9. Find the volume of the solid obtained by rotating the region enclosed by $y = \sqrt[3]{x}, x = 1, x = 2$ and $y = 0$ *about the y-axis.*

Problem 10. Find the volume of the solid obtained by revolving the region in the first quadrant $(x, y \geq 0)$ enclosed by the x- and y-axes and the graph of $y = e^{-x}$ about the the x-axis.

Problem 11. Determine whether the sequence

$$a_n = \frac{(n-1)(n+1)n}{n^2 + 1 - 3n^3}$$

converges or diverges. If it converges, find its limit.

Problem 12. Find an example of each of the following situations.

(a) Two sequences (a_n) and (b_n) so that $(a_n + b_n)$ converges but at least one of (a_n) or (b_n) does not converge.

(b) Two sequences (a_n) and (b_n) so that $(a_n b_n)$ converges but at least one of (a_n) or (b_n) does not converge.

Problem 13. Let A be the region in the first quadrant $(x, y \geq 0)$ enclosed by $y = x$ and $y = x^3$. Sketch the region below. Then set up two integrals expressing the area of A: one dx integral and one dy integral. Finally, evaluate either of your integrals to determine the area of A.