Problem 1. Find the minimal polynomial of $\alpha = \sqrt{2} + \sqrt[2]{3}$ over $\mathbb{Q}$.

Problem 2. Decide whether each polynomial is irreducible. If it's reducible, find its irreducible factors.

(a) $x^7 + 11x^3 - 33x + 22$ over $\mathbb{Q}$

(b) $x^7 + 11x^3 - 33x + 22$ over $\mathbb{F}_{11}$

(c) $x^3 + 7x + 7^{2022} + 2$ over $\mathbb{Q}$

(d) $x^4 + 4$ over $\mathbb{F}_5$

Problem 3. Prove that every ring homomorphism $\varphi : \mathbb{Q} \to \mathbb{Q}$ is the identity.

Problem 4. Let $I = (f, g)$ be the ideal in $\mathbb{Q}[x]$ generated by $f(x) = x^4 - x^3 + x - 1$ and $g(x) = x^4 + x^2 + 1$.

(a) Is I principal? Why or why not?

(b) If I is principal, find an $h(x) \in \mathbb{Q}[x]$ such that $I = (h)$.

Problem 5. Let A and B be rings. Prove that $(A \times B)^\times \cong A^\times \times B^\times$ as groups.

Problem 6. Which of the following complex numbers can be constructed using a ruler and compass in the complex plane? If possible, show how to construct the constructible ones.

(a) $\sqrt{2}$

(b) $\sqrt{2}+\sqrt{3}$

(c) $\sqrt[5]{5}$

(d) $2+\sqrt{i}$

(e) $\cos\left(\frac{\pi}{16}\right) - i\sin\left(\frac{\pi}{16}\right)$