This induces an extension of function fields $L/K := k(Y)/k(X)$.

By Artin-Schreier-Witt theory,

$$L = K(\wp^{-1}(\underline{f}))$$

for some Witt vector-valued rational function f on X

$$f : X \dashrightarrow \mathbb{W}_n$$

This misses important information at a finite set of points of X (= poles of f).

Garuti's solution: just like rational functions $f : X \dashrightarrow \mathbb{A}^1$ can be viewed as regular maps $X \longrightarrow \mathbb{P}^1$, there exists a projective variety

$$\overline{\mathbb{W}_n} \supseteq \mathbb{W}_n$$

such that rational functions $f : X \dashrightarrow \mathbb{W}_n$ correspond to regular maps $X \longrightarrow \mathbb{W}_n$.

Even nicer: this compactification is equivariant with respect to

$$\mathbb{W}_n \curvearrowright \mathbb{W}_n$$

$$\underline{a} \cdot \underline{x} = \underline{a} + \underline{x}$$

Ramification data: invariants of the cover $Y \to X$, which generalize the ramification jump in the $\mathbb{Z}/p\mathbb{Z}$ case can be determined

$\mathbb{Z}/p\mathbb{Z}$ case, can be determined

explicitly :

$$\begin{tikzcd} Y \arrow[r] \arrow[d, "\mathbb{Z}/p^n\mathbb{Z}"'] & \overline{\mathbb{W}}_n \arrow[d, "\Psi_n \supseteq \wp"] \\ X \arrow[r, "f"] & \overline{\mathbb{W}}_n \supseteq \end{tikzcd}$$

$B_n = \overline{\mathbb{W}}_n \setminus \mathbb{W}_n$

$\wp \underline{x} - \underline{f}$

$\mathrm{ord}_x(f^* B_n) =$

last ram. jump at x

$\boxed{\text{Ex}}$ The Artin–Schreier case

$$\begin{tikzcd} y^p - y = f \quad Y \arrow[r] \arrow[d, "\mathbb{Z}/p\mathbb{Z}"'] & \mathbb{P}^1 \arrow[d] \\ {} & {} \end{tikzcd}$$

$$\begin{array}{ccc} {\scriptstyle \mathbb{Z}/p\mathbb{Z}}\downarrow & & \downarrow \\ X & \xrightarrow{f} & \mathbb{P}^1 \end{array}$$

$\mathrm{ord}_x(f)$ $\longleftarrow$ $\infty = B$,

$=$ ord of the pole of f at x

$=$ ram. jump at x.

Moduli Stacks of Covers

Suppose we want to study these covers of curves $Y \longrightarrow X$ in families :

- a **family of curves** over a scheme T (or a **T-curve**) is a flat morphism

$$X \longrightarrow T$$

such that each fibre X_t is a (smooth projective) curve

- a **family of covers of curves** (or a **cover of T-curves**) is a commutative triangle

$$Y \longrightarrow X$$

$$\begin{tikzcd} Y \arrow[rr] \arrow[dr] & & X \arrow[dl] \\ & T & \end{tikzcd}$$

where all the fibres $Y_t \to X_t$ are covers of curves.

In algebraic geometry, often when we want to study such objects, they arise as the T-points of a moduli space, e.g.

- $\mathcal{M}_g$ = smooth genus g curves

- $\mathcal{H}_g$ = hyperelliptic curves of gen. g

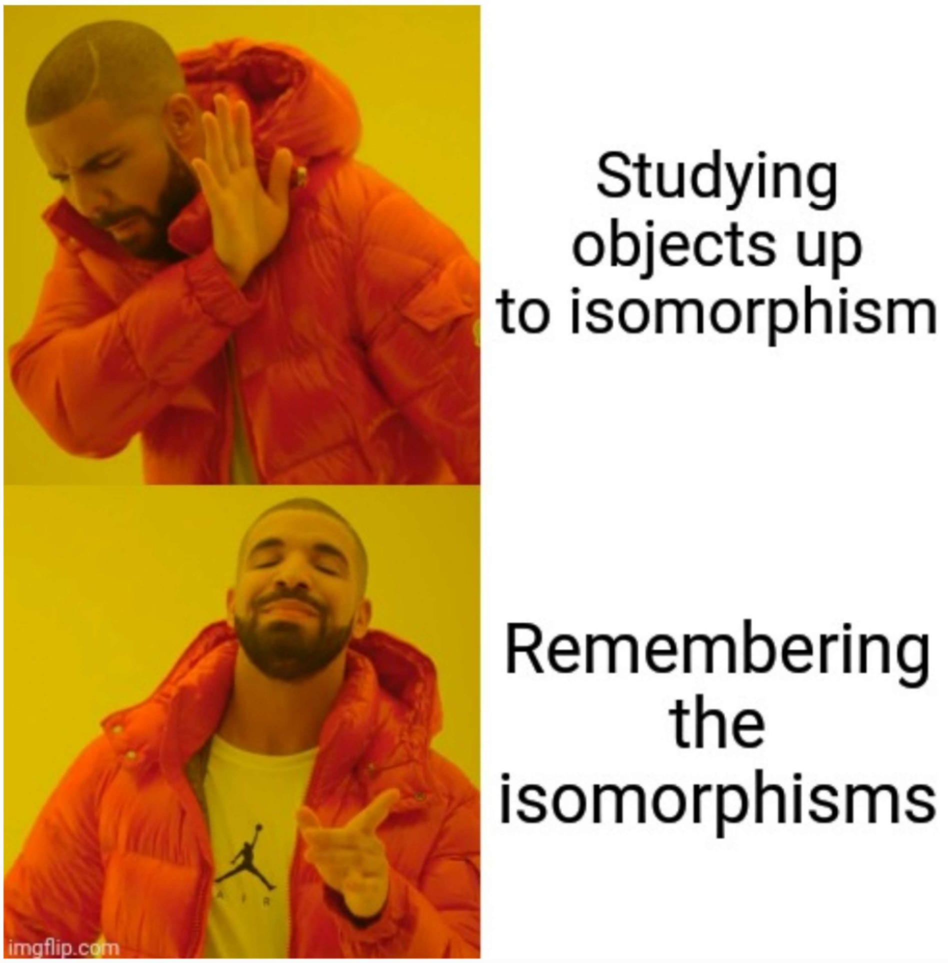
- $\mathcal{AS}_g$ = Artin-Schreier curves

  $\mathbb{Z}/p \hookrightarrow \mathrm{Aut}(X)$

- $\mathcal{AS}cov_g$ = AS-covers

$$\mathbb{Z}/p \curvearrowright \begin{array}{c} Y \\ \downarrow \\ X (= \mathbb{P}^1) \end{array}$$

We can further stratify these spaces by specifing things like the p-rank, a-number, ramification invariants, etc.



(moduli spaces)

(moduli stacks)

My work: Say $X = \mathbb{P}^1$ for simplicity. Let

$$\mathrm{AScov}_g^j \subseteq \mathrm{AScov}_g$$

denote the substack of AS covers $Y \xrightarrow{\varphi} \mathbb{P}^1$ where

- $\text{genus}(Y) = g$
- φ is branched at ∞ with ramification jump j

Then points of $\underline{AScov_g^j(T)}$ are classified by maps to a weighted projective line $\mathbb{P}(1,j)$:

$$\begin{array}{ccccc} Y & \longrightarrow & \mathbb{P}(1,j) & & \overline{\mathbb{W}}_1 \\ \mathbb{Z}/p\mathbb{Z} \downarrow & \text{ét. loc.} & \downarrow \Psi_1 & & \downarrow \Psi_1 \end{array}$$

$$\begin{tikzcd} \mathbb{P}^1 \arrow[r] \arrow[rr, bend right] & \mathbb{P}(1,j) \arrow[r] & \mathbb{P}^1 = \overline{\mathbb{M}}_1 \end{tikzcd}$$

Work in progress (joint with L. Herr, V. Thatte)

Viewing $\mathbb{P}^1 = \overline{\mathbb{M}}_1$, we can extend this theory to ASW covers as follows: consider the moduli stack

$$\mathrm{ASWcov}_g^{j_1,\dots,j_n} \subseteq \mathrm{ASWcov}_g$$

Then points of $\mathcal{AS}\mathcal{W}_g^{j_1,\dots,j_n}$ are classified by maps to a stacky compactification of $\mathbb{W}_n$:

$$\begin{tikzcd}
Y \arrow[r] \arrow[d, "\mathbb{Z}/p^n\mathbb{Z}"'] & \overline{\mathbb{W}_n(j_1,\dots,j_n)} \arrow[d] \\
\mathbb{P}^1 \arrow[r] & \overline{\mathbb{W}_n(j_1,\dots,j_n)}
\end{tikzcd}$$

Notes: • $\overline{\mathbb{W}}_n(j_1, \ldots, j_n)$ is an iterated root stack over $\overline{\mathbb{W}}_n$

• $\overline{\mathbb{W}}_1(j) = \mathbb{P}(1, j)$

• These can be used to study the local structure of (Deligne-Mumford) stacks themselves in characteristic $p > 0$.

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Artin-Schreier Root Stacks

Andrew Kobin

THANK YOU!

QUESTIONS?